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Role of Modified Chaplygin Gas in Bianchi Type-I Universe

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Abstract We have studied the evolution of a homogeneous, anisotropic universe given by a Bianchi type-I cosmological model with modified Chaplygin gas. We have assumed that the equation of state of this modified model is valid from the radiation era to the ΛCMD model. We have used state-finder parameters in characterizing different phase of the model.

Keywords Cosmological universe · Modified Chaplygin gas · State-finder parameter · Cosmological parameter

1 Introduction

Recent observations of the luminosity of type Ia supernovae indicate [1, 2] an accelerated expansion of the universe and lead to the search for a new type of matter which violates the strong energy condition i.e. $\rho + 3p < 0$ is satisfied. The matter content responsible for such a condition to be satisfied at a certain stage of evolution of the universe is referred to as dark energy. There are different candidates to play the role of the dark energy. The type of dark energy represented by a scalar field is often called quintessence. The transition from a universe filled with matter to an exponentially expanding universe does not necessarily require the presence of a scalar field as the only alternative. In particular, one can try another alternative by using an exotic type of fluid—the so called Chaplygin gas which obeys an equation of state such as $p = -B/\rho$ [3, 4], where p and ρ are respectively the pressure and energy density and B is positive constant. Subsequently the above equation was modified to the form $p = -B/\rho^{\alpha}$ with $0 < \alpha \leq 1$. This model gives the cosmological evolution from initial dustlike matter to an asymptotic cosmological constant and a fluid obeying an equation of state $p = \alpha\rho$. This generalized model has been studied previously [5–7].

The statefinder diagnostic along with future SNAP observations may be used to discriminate between different dark energy models. The statefinder diagnostic pair is constructed from the scale factor $a(t)$ and its derivatives up to the third order as follows

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(Sahni et al. [8])

$$r = \frac{\dddot{a}}{aH^3} \quad \text{and} \quad s = \frac{r-1}{3(q-\frac{1}{2})} \tag{1.1}$$

where H and $q(=-\frac{a\ddot{a}}{\dot{a}^2})$ are the Hubble parameters are dimensionless and allow us to characterize the properties of dark energy in a model independent manner. The parameter r forms the next step in the hierarchy of geometrical cosmological parameters after H and q.

In our present work, we consider a more general modified Chaplygin gas obeying an equation of state [9]

$$p = A\rho - \frac{B}{\rho^\alpha} \quad \text{with } 0 \le \alpha \le 1. \tag{1.2}$$

This equation of state shows a radiation era (when $A = 1/3$) at one extreme (when the scale factor $a(t)$ is vanishing small) and a ΛCMD model at the other extreme (when the scale factor $a(t)$ is infinitely large) At all stages it shows a mixture. Also in between there is one stage when the pressure vanishes and the matter content is equivalent to pure dust. We have further described this particular cosmological model from the field theoretical point of view by introducing a scalar field ϕ and a self-interacting potential $V(\phi)$ with the effective Lagrangian

$$L_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{1.3}$$

In the paper of Gorini et al. [5, 6], it has been shown that the simple flat Friedmann model with Chaplygin gas can equivalently be described in terms of a homogeneous minimally coupled scalar field ϕ. In this case FRW equations for Chaplygin gas fit into Barrow's scheme [10]. Following Barrow [11], Kamenshchik et al. [3, 4, 12] have obtained homogeneous scalar field $\phi(t)$ and a potential $V(\phi)$ to describe Chaplygin cosmology. Debnath et al. [13] have studied the role of modified Chaplygin gas in accelerated universe. In the present work we have studied the influence of modified Chaplygin gas in Bianchi type-I universe.

2 Modified Chaplygin Gas in Bianchi Type-I Universe

The metric of a homogeneous and anisotropic universe in the Bianchi type-I universe is

$$ds^2 = dt^2 - {a_1}^2dx^2 - {a_2}^2dy^2 - {a_3}^2dz^2 \tag{2.1}$$

where the metric functions a_1, a_2, a_3 are functions of t only.

The Einstein field equations for the metric (2.1) are written in the form

$$\frac{\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3a_1} = \rho, \tag{2.2}$$

$$\frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_2\dot{a}_3}{a_2a_3} = -p, \tag{2.3}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_3}{a_3} + \frac{\dot{a}_1\dot{a}_3}{a_1a_3} = -p, \tag{2.4}$$

$$\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\dot{a}_1\dot{a}_2}{a_1a_2} = -p \tag{2.5}$$

where ρ and p are the energy density and pressure, respectively (choosing $8\pi G = c = 1$).

We define

$$V = a_1 a_2 a_3. \tag{2.6}$$

Subtracting (2.3) from (2.4) [14–16], we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\left(\frac{\dot{a}_1}{a_1} + \frac{\dot{a}_2}{a_2} + \frac{\dot{a}_3}{a_3}\right) = 0. \tag{2.7}$$

Using (2.6) in (2.7), we get

$$\frac{d}{dt}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right) + \left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)\frac{\dot{V}}{V} = 0. \tag{2.8}$$

Integrating the above equation, we get

$$\frac{a_1}{a_2} = d_1 \exp\left(x_1 \int \frac{dt}{V}\right), \quad d_1 = \text{constant}, \ x_1 = \text{constant}. \tag{2.9}$$

By subtracting (2.5) from (2.3) and (2.4) from (2.5), we can obtain similarly

$$\frac{a_1}{a_3} = d_2 \exp\left(x_2 \int \frac{dt}{V}\right), \tag{2.10}$$

$$\frac{a_2}{a_3} = d_3 \exp\left(x_3 \int \frac{dt}{V}\right) \tag{2.11}$$

where d_2, d_3, x_2, x_3 are integration constants.

In view of (2.6) we find the following relation between the constants $d_1, d_2, d_3, x_1, x_2, x_3$

$$d_2 = d_1 d_3, \qquad x_2 = x_1 + x_3.$$

Finally from (2.9), (2.10) and (2.11), we write $a_1(t)$, $a_2(t)$, and $a_3(t)$ in the explicit form.

$$a_1(t) = D_1 V^{1/3} \exp\left(X_1 \int \frac{dt}{V(t)}\right), \tag{2.12}$$

$$a_2(t) = D_2 V^{1/3} \exp\left(X_2 \int \frac{dt}{V(t)}\right), \tag{2.13}$$

$$a_3(t) = D_3 V^{1/3} \exp\left(X_3 \int \frac{dt}{V(t)}\right), \tag{2.14}$$

where D_i $(i = 1, 2, 3)$ and X_i $(i = 1, 2, 3)$ satisfy the relation $D_1 D_2 D_3 = 1$ and $X_1 + X_2 + X_3 = 0$.

Now, adding (2.3), (2.4), (2.5) and three times (2.2), we get

$$\left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1}\right) = \frac{3\kappa}{2}(\rho - p). \tag{2.15}$$

From (2.6) we have

$$\frac{\ddot{V}}{V} = \left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} + \frac{\ddot{a}_3}{a_3}\right) + 2\left(\frac{\dot{a}_1\dot{a}_2}{a_1 a_2} + \frac{\dot{a}_2\dot{a}_3}{a_2 a_3} + \frac{\dot{a}_3\dot{a}_1}{a_3 a_1}\right). \tag{2.16}$$

From (2.15) and (2.16) we obtain

$$\frac{\ddot{V}}{V} = \frac{3}{2}(\rho - p). \tag{2.17}$$

The energy conservation equation is

$$\dot{\rho} + \frac{\dot{V}}{V}(\rho + p) = 0. \tag{2.18}$$

From (1.2) and (2.18), we have

$$\rho = \left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{2.19}$$

where C is an arbitrary integration constant.

Now for a small value of the scale factors $a_1(t)$, $a_2(t)$ and $a_3(t)$, we have

$$\rho \cong \frac{C^{\frac{1}{1+\alpha}}}{V^{(1+A)}} \tag{2.20}$$

which is very large and corresponds to the universe dominated by an equation of state $p = A\rho$.

Then from (2.17), we have

$$\frac{\ddot{V}}{V} = \frac{3}{2}(1-A)\frac{C^{\frac{1}{1+\alpha}}}{V^{1+A}} \tag{2.21}$$

which gives

$$\int \frac{dV}{\sqrt{3C^{\frac{1}{1+\alpha}}V^{(1-A)} + C_1}} = t - t_0 \tag{2.22}$$

where $C_1 \equiv$ integration constant.

For $C_1 = 0$, (2.22) gives

$$V = \left[\frac{3(A+1)}{4}\right]^{\frac{1}{A+1}} C^{\frac{1}{(1+\alpha)(1+A)}} t^{\frac{2}{A+1}}. \tag{2.23}$$

From (2.12), (2.13), (2.14) and (2.23), we get

$$\begin{aligned} a_1(t) = D_1 \left[\frac{3(A+1)}{4}\right]^{\frac{1}{3(A+1)}} C^{\frac{1}{3(1+\alpha)(1+A)}} t^{\frac{2}{3(A+1)}} \\ \times \exp\left[X_1 \left\{\frac{4}{3(A+1)C^{1/1+\alpha}}\right\}^{\frac{1}{1+A}} \left(\frac{A+1}{A-1}\right) t^{(\frac{A-1}{A+1})}\right], \end{aligned} \tag{2.24}$$

$$a_2(t) = D_2 \left[\frac{3(A+1)}{4}\right]^{\frac{1}{3(A+1)}} C^{\frac{1}{3(1+\alpha)(1+A)}} t^{\frac{2}{3(A+1)}}$$

$$\times \exp\left[X_2\left\{\frac{4}{3(A+1)C^{1/1+\alpha}}\right\}^{\frac{1}{1+A}}\left(\frac{A+1}{A-1}\right)t^{\left(\frac{A-1}{A+1}\right)}\right], \tag{2.25}$$

$$a_3(t) = D_3\left[\frac{3(A+1)}{4}\right]^{\frac{1}{3(A+1)}} C^{\frac{1}{3(1+\alpha)(1+A)}}\, t^{\frac{2}{3(A+1)}}$$

$$\times \exp\left[X_3\left\{\frac{4}{3(A+1)C^{1/1+\alpha}}\right\}^{\frac{1}{1+A}}\left(\frac{A+1}{A-1}\right)t^{\left(\frac{A-1}{A+1}\right)}\right]. \tag{2.26}$$

From (2.20) and (2.23) we get

$$\rho = \frac{4}{3(A+1)^2}\frac{1}{t^2} \tag{2.27}$$

and

$$p = \frac{4A}{3(A+1)^2}\frac{1}{t^2}. \tag{2.28}$$

The physical quantities of observational interest in cosmology are the expansion scalar θ, the mean anisotropy parameter A, the shear scalar σ^2 and the deceleration parameter q. They are defined as [17]

$$\theta = 3H, \tag{2.29}$$

$$A = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{2.30}$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^{3} {H_i}^2 - 3H^2\right) = \frac{3}{2}AH^2, \tag{2.31}$$

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1. \tag{2.32}$$

From (2.29)–(2.32), we obtain

$$\theta = \left(\frac{2}{A+1}\right)\frac{1}{t}, \tag{2.33}$$

$$A = \frac{3X^2(A+1)^2}{4}\left\{\frac{4}{3(A+1)C^{\frac{1}{1+\alpha}}}\right\}^{\frac{1}{1+A}} t^{2\left(\frac{A-1}{A+1}\right)}, \tag{2.34}$$

$$\sigma^2 = \frac{X^2}{2}\left\{\frac{4}{3(A+1)C^{\frac{1}{1+\alpha}}}\right\}^{\frac{1}{1+A}} t^{-\left(\frac{4}{A+1}\right)}, \tag{2.35}$$

$$q = \frac{1}{2}(1+3A) \tag{2.36}$$

where $X^2 \equiv {X_1}^2 + {X_2}^2 + {X_3}^2$.

From (1.1) and (2.23)

$$r = \frac{9}{2}A(A+1) + 1 \tag{2.37}$$

and

$$s = 2(A+1). \tag{2.38}$$

For a large value of the scale factors $a_1(t), a_2(t), a_3(t)$. we obtain

$$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} \tag{2.39}$$

and

$$p \approx -\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}}. \tag{2.40}$$

Then from (2.17), we have

$$\frac{\ddot{V}}{V} = 3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} \tag{2.41}$$

which gives

$$V = \sqrt{\frac{2C_1}{3}}\left(\frac{1+A}{B}\right)^{\frac{1}{2(1+\alpha)}} \sinh\left\{3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} t + C_2\right\} \tag{2.42}$$

where C_1 and C_2 are integration constants.
From (2.12), (2.13), (2.14) and (2.42), we get

$$a_1(t) = D_1[K_1 \sinh(K_2 t + C_2)]^{1/3} \exp\left[-\frac{X_1}{K_1 K_2} \cot^{-1}\{\cosh(K_2 t + C_2)\}\right], \tag{2.43}$$

$$a_2(t) = D_2[K_1 \sinh(K_2 t + C_2)]^{1/3} \exp\left[-\frac{X_2}{K_1 K_2} \cot^{-1}\{\cosh(K_2 t + C_2)\}\right], \tag{2.44}$$

$$a_3(t) = D_3[K_1 \sinh(K_2 t + C_2)]^{1/3} \exp\left[-\frac{X_3}{K_1 K_2} \cot^{-1}\{\cosh(K_2 t + C_2)\}\right] \tag{2.45}$$

where $K_1 \equiv \sqrt{\frac{2C_1}{3}}(\frac{1+A}{B})^{\frac{1}{2(1+\alpha)}}$ and $K_2 \equiv 3(\frac{B}{1+A})^{\frac{1}{1+\alpha}}$ are constants.
The physical quantities of observational interest in cosmology are

$$\theta = K_2 \coth(K_2 t + C_2), \tag{2.46}$$

$$A = \frac{3X^2}{{K_2}^4} \frac{\sinh^4(K_2 t + C_2)}{\cosh^2(K_2 t + C_2)[1 + \cosh^2(K_2 t + C_2)]^2}, \tag{2.47}$$

$$\sigma^2 = \frac{X^2}{2} \frac{\sinh^2(K_2 t + C_2)}{[1 + \cosh^2(K_2 t + C_2)]^2}, \tag{2.48}$$

$$q = \frac{3}{\cosh^2(K_2 t + C_2)} - 1 \tag{2.49}$$

where $X^2 \equiv {X_1}^2 + {X_2}^2 + {X_3}^2$.

From (1.1) and (2.42), we get

$$r = 9\mathrm{sech}^2\left\{3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} t + C_2\right\} \tag{2.50}$$

and

$$s = \frac{2}{1-\sinh^2\{3(\frac{B}{1+A})^{\frac{1}{1+\alpha}} t + C_2\}}. \tag{2.51}$$

Considering now the subleading terms in (2.19) at large values of a_1, a_2 and a_3. We can obtain the following expressions for the energy density and pressure.

$$\rho \approx \left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} + \frac{C}{1+\alpha}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}} V^{-(1+\alpha)(1+A)}, \tag{2.52}$$

$$p \approx -\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} + \frac{C}{1+\alpha}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}} \{\alpha + (1+\alpha)A\}\, V^{-(1+\alpha)(1+A)}. \tag{2.53}$$

From (2.16) we have

$$\ddot{V} = 3\left(\frac{B}{1+A}\right)^{\frac{1}{1+\alpha}} V + \frac{3}{2}\frac{C}{1+\alpha}\left(\frac{1+A}{B}\right)^{\frac{1}{1+\alpha}} \{1-\alpha + (1+\alpha)A\}\, V^{1-(1+\alpha)(1+A)}. \tag{2.54}$$

On integration, which gives

$$\int \frac{dV}{\sqrt{K_3 V^2 + K_4 V^{(1-A-\alpha-\alpha A)} + K_5}} = t \tag{2.55}$$

where $K_3 \equiv 3(\frac{B}{1+A})^{\frac{1}{1+\alpha}} = K_2$, $K_4 \equiv \frac{3}{2}\frac{C}{1+\alpha}(\frac{1+A}{B})^{\frac{1}{1+\alpha}}(\frac{1-\alpha+A+\alpha A}{1-A-\alpha-\alpha A})$ and K_5 is integration constant.

Equations (2.52) and (2.53) describe the mixture of a cosmological constant equal to $(\frac{B}{1+A})^{\frac{1}{1+\alpha}}$ with matter whose equation of state is given by

$$p = \{\alpha + (1+\alpha)A\}\,\rho \tag{2.56}$$

which for a pure Chaplygin gas reduces to $p = \alpha\rho$.

Now we consider the energy density and pressure corresponding to a scalar field ϕ having a self-interacting potential $V(\phi)$. The energy ρ_ϕ and p_ϕ are defined as

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho = \left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} \tag{2.57}$$

and

$$\begin{aligned} p_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) = A\rho - \frac{B}{\rho^\alpha} \\ &= A\left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{-\alpha}{1+\alpha}}. \end{aligned} \tag{2.58}$$

From (2.57) and (2.58), we have

$$\dot{\phi}^2 = (1+A)\left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} - B\left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{-\alpha}{1+\alpha}} \tag{2.59}$$

and

$$V(\phi) = \frac{1}{2}(1-A)\left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{1}{1+\alpha}} + \frac{B}{2}\left[\frac{B}{1+A} + \frac{C}{V^{(1+A)(1+\alpha)}}\right]^{\frac{-\alpha}{1+\alpha}}. \tag{2.60}$$

For $A = 1$ and $\alpha = 1$, then (2.55) gives

$$V = \left[\sqrt{\frac{C + K_5{}^2/9}{2B}} \cosh\left\{2\sqrt{3}\left(\frac{B}{2}\right)^{1/4} t\right\} - \frac{K_5}{3\sqrt{2B}}\right]^{1/2}. \tag{2.61}$$

From (2.12)–(2.14) and (2.61), we obtain

$$\begin{aligned} a_1(t) &= D_1[A_1 \cosh(A_2 t) - A_3]^{1/6} \\ &\quad \times \exp\left[-\frac{2X_1 i\sqrt{\frac{A_1\cosh(A_2t)}{A_1 - A_3}}\, EllipticF[\frac{iA_2 t}{2}, \frac{2A_1}{A_1 - A_3}]}{B\sqrt{A_1\cosh(A_2 t) - A_3}}\right], \end{aligned} \tag{2.62}$$

$$\begin{aligned} a_2(t) &= D_2[A_1 \cosh(A_2 t) - A_3]^{1/6} \\ &\quad \times \exp\left[-\frac{2X_2 i\sqrt{\frac{A_1\cosh(A_2t)}{A_1 - A_3}}\, EllipticF[\frac{iA_2 t}{2}, \frac{2A_1}{A_1 - A_3}]}{B\sqrt{A_1\cosh(A_2 t) - A_3}}\right], \end{aligned} \tag{2.63}$$

$$\begin{aligned} a_3(t) &= D_3[A_1 \cosh(A_2 t) - A_3]^{1/6} \\ &\quad \times \exp\left[-\frac{2X_3 i\sqrt{\frac{A_1\cosh(A_2t)}{A_1 - A_3}}\, EllipticF[\frac{iA_2 t}{2}, \frac{2A_1}{A_1 - A_3}]}{B\sqrt{A_1\cosh(A_2 t) - A_3}}\right], \end{aligned} \tag{2.64}$$

where $A_1 \equiv \sqrt{\frac{C+K_5{}^2/9}{2B}}$, $A_2 \equiv 2\sqrt{3}(\frac{B}{2})^{1/4}$ and $A_3 \equiv \frac{K_5}{3\sqrt{2B}}$.

The physical quantities are

$$\theta = \frac{A_1 A_2 \sinh(A_2 t)}{2\{A_1\cosh(A_2 t) - A_3\}}, \tag{2.65}$$

$$q = 6\,\mathrm{cosech}^2(A_2 t)\left[\mathrm{cosech}(A_2 t) - \left(\frac{A_3}{A_1}\coth(A_2 t)\right)\right] - 1. \tag{2.66}$$

From (1.1) and (2.61), we get

$$r = 9\left(\frac{A_3{}^2}{A_1{}^2} - 1\right)\mathrm{cosech}^2(A_2 t) + 1 \tag{2.67}$$

and

$$s = \frac{2(\frac{A_3{}^2}{A_1{}^2} - 1)}{\{1 - 2\cosh(A_2 t)\}\{A_1\cosh(A_2 t) - A_3\}}. \tag{2.68}$$

3 Conclusion

The evolution of a homogeneous, anisotropic universe given by a Bianchi type-I cosmological model with modified Chaplygin gas. We have assumed that the equation of state of this modified model is valid from the radiation era to the ΛCMD model. We have used state-finder parameters in characterizing different phase of the model.

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